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Distended Topologically Massive Electrodynamics

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Dedicated to the memory of Wolfgang Kummer, intrepid explorer of (a somewhat) lower dimension. To appear in Kummer memorial volume.

Abstract

We extend topologically massive electrodynamics, both by adding a higher derivative action to cast the entire three-term model in Chern-Simons (CS) form, and by embedding it in an AdS background. It can then be written as the sum of two CS terms, one of which vanishes at the "chiral" point, in analogy with its gravitational topologically massive counterpart. Separately we treat pure CS electrodynamics plus Einstein gravity interacting with point sources. The gravity/vector field equations decouple; their solutions are the familiar exterior "conical" metric and vector potentials.

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1 Introduction

Theories involving Chern-Simons (CS) terms have remained popular ever since their introduction, in both gravitational and vector incarnations, over a quarter-century ago [1]. Most recently, there has been a great deal of work on extension of the original models from flat to AntideSitter (AdS) backgrounds [2,3] Separately, standard topologically massive electrodynamics (TME) has been augmented by higher derivative, but CS-like, terms [4]. Here we combine these two generalizations to express TME as a "2-CS" chiral sum.

Our second topic is "pure CS" - combined Einstein plus vector CS actions - in presence of point masses and charges. In both cases, the field equations exhibit a field-current identity. Further, the two field sectors decouple, so the total system remains soluble. Accordingly, the resulting metric and vector potential have conical structure.

2 E(xtended)TME

Gravitational (tensor) and electrodynamical (vector) field models are often both similar and different; this is also true of their CS properties. The most relevant difference is that the gravitational CS actions is of third derivative order - higher than the Einstein action - whereas it is the opposite for vectors, whose CS term is first order, lower than the Maxwell action's. This is reflected in constructing their "pure CS" extensions, requiring respectively cosmological/higher derivative additions to the topologically massive two-term models. However, even for TME, as we shall see, adding a cosmological background simulates TMG in this context. For simplicity, we consider only abelian TME here.

We begin with the vector CS action,

$$I_{CS}(B) = \int d^3x\, \epsilon^{\mu\nu\alpha} B_\mu \partial_\nu B_\alpha \,. \tag{2.1}$$

The resulting field equation,

$$F^\mu(B) \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha} F_{\nu\alpha}(B) = 0 \tag{2.2}$$

states that field space is "flat," with a pure gauge vector potential. Next, generalize B_μ to be a combination

$$B^\pm_\mu(A) \equiv m^{-1/2} F_\mu(A) \pm m^{1/2} A_\mu \tag{2.3}$$

of the fundamental variable A_μ. The parameter m has dimensions of mass, needed to give A_μ its canonical dimension. We have also allowed for separate combinations $B_\pm$ which could be further generalized by allowing for two separate mass values, $m_\pm$. Our conventions are $(-++)$ signature, $\epsilon^{012} = 1$; the background is (initially) flat.

The action (2.1) with $B(A)$ as in (2.3) consists of three terms,

$$\begin{aligned} I^\pm_{CS}\left(B^\pm(A)\right) &= m^{-1}\int d^3x\, \epsilon^{\mu\nu\alpha}\left\{F_\mu(A)\partial_\nu F_\alpha(A) + m^2 A_\mu \partial_\nu A_\alpha\right\} \pm \int d^3x\, F^2_{\mu\nu}(A) \\ &\equiv \left\{m^{-1} I_{ECS} + m I_{CS}\right\} \mp 4 I_{MAX}\,. \end{aligned} \tag{2.4}$$

This result confirms for spin 1 that "everything is CS" in $D = 3$; even the Maxwell action is the difference of two CS terms. The two-mass generalization would permit even more flexibility in the relative coefficients.

The dynamics of the various three-term models in (2.4) was analyzed in [4], whose results we summarize for completeness. Pure I_{ECS} leads to a null-propagating field strength, $\Box F_{\mu\nu} = 0$, and hence does allow excitations, distinct from Maxwell's where $F^{\mu\nu}$ is of course also divergence-free. This model differs from pure I_{CS} in not being topological: for example, its action is metric - dependent and more fundamentally, it shows no interesting large gauge behavior, owing to the pure field-strength dependence, even in the nonabelian version. The combination $I_{ECS} + I_{MAX}$ also differs from the original TME: it contains a massive ghost excitation as well as the photon mode. Combining $I_{ECS} + I_{CS}$ does not add further excitations to that of I_{ECS} alone: instead, the field strength now propagates massively. Finally, the full three-term action depends on the two relative internal coefficients, and there are in general three masses, though there can be a degeneracy for suitable tuning. In all cases a ghost is unavoidable.

The above results are easily checked explicitly from the field equations, with the usual decompositions of the potentials into invariant, pure gauge and constraint components: since each term is separately gauge invariant, all excitations are as well, and depend only on the transverse vector potential, effectively the indexless scalar in $A_i^T = \epsilon^{ij}\partial_j S$.

3 C(osmological)ETME

We now introduce a nontrivial - AdS - gravitational background. The relevant aspect of this generalization is the appearance of a second dimensional parameter, the cosmological constant, $\Lambda \equiv -\ell^{-2}$.

Let us modify our previous discussion to follow the gravitational "2-CS" formulation of [2]*. There, the variable corresponding to $B^{\pm}$ of (2.3) is a very similar combination of $B^{\pm}_{\mu}$, namely $\omega_{\mu}^{ab}(e) \pm \ell^{-1}\epsilon^{abc}e_{\mu c}$, where $\omega(e)$ is the spin connection constructed from the dreibein $e_{\mu c}$. Note the required inverse length, which we mostly set to unity. In this fashion, we get two gravitational CS combinations

$$I_{\pm}[\omega(e) \pm e] = I_{GCS}[\omega(e)] \mp I_{GR}[e], \tag{3.1}$$

where $I_{GCS} \sim \frac{1}{2}\int d^3x(\epsilon\omega\partial\omega + \ldots)$, is the (third derivative) gravitational CS term, I_{GR} is the Einstein action including the cosmological term (proportional to Λ) but with the "wrong" sign required by TMG to ensure ghost freedom. To construct the cosmological topologically massive action, a mass parameter m—distinct from ℓ^{-1}—is introduced by hand to yield, from (3.1),

$$\begin{aligned} 2I_{CTMG} &= (1 + m^{-1})I^{-} + (1 - m^{-1})I^{+} \\ &= -\int d^3x\,\sqrt{-g}(R - \Lambda) + m^{-1}I_{GCS} \end{aligned} \tag{3.2}$$

*This was also found by D. Grumiller and R. Jackiw (unpublished.)

in Planck units. This CS doublet degenerates to a single term at either "chiral" value $m = \pm 1$.

Returning to our vector case, we define the extended variable to be the $m = \ell^{-1}$ value of (2.3),

$$B^{\pm}_{\mu}(A) = f_{\mu}(A) \pm A_{\mu} \,. \tag{3.3}$$

[The other effect of the nontrivial background, say $g_{\mu\nu} = \phi^2 \eta_{\mu\nu}$, is that $f_{\mu}(A)$ is here a covariant vector, like A_{μ} so it acquires a factor ϕ^{-1} . Hence I_{ECS} is scaled by ϕ^{-2}, while $I_{MAX} \sim \phi^{-1}$ and of course I_{CS} is metric-independent. These extra factors are not directly relevant to our discussion.] The analog of (2.4) is simply obtained by replacing m by ℓ^{-1} there. Consequently, the CETME action is the combination

$$8I = (4m\ell - 1)I^{+} + (4m\ell + 1)I^{-} \tag{3.4}$$

where m is the mass parameter of TME, and we have restored ℓ explicitly. This parallels the gravitational form (3.2) except for the dimensionally dictated $m \rightarrow \frac{1}{m}$ there. This is the 3-term analog of TMG, and all three terms must be present. The $m \rightarrow 0$ limit is of course Maxwell, but ordinary 2-term TME is obtained only in the singular $\ell \rightarrow 0$ limit, while for gravity, it is the infinite mass limit that yields the (cosmological) Einstein action.

At the chiral points $4m\ell = \pm 1$, one of the actions vanishes, exactly as for chiral gravity. The physics of ordinary two-term CTME at the chiral point is laid out in [2], where it is shown to be in one-to-one correspondence with linearized CTMG at the latter's chiral point.

4 Sources

So far, we have studied our models in a source-free context. We now include sources, in a particular, "pure CS", gravity plus CS context.

It is instructive to first analyze the relevant similarities to - and differences from - the gravitational case. Recall that for spin two, the highest, third derivative, term is the gravitational CS action; its variation is the Cotton - conformal curvature - tensor, whose vanishing implies the metric is conformally flat. The Einstein action instead, effectively resembles that of pure vector CS: in both cases, their variations are the respective "curvature terms," whose vanishing implies field flatness. [The Maxwell term has no gravitational analog since it does describe a single physical excitation.] It is therefore really the Einstein and vector CS terms that corresponded most closely in the two systems. In each case, there is a field - current identity, respectively

$$G_{\mu\nu} = T_{\mu\nu} \tag{4.1}$$

$$F^{\mu} = j^{\mu} \tag{4.2}$$

where the Einstein tensor in (4.1) equivalent (being its double dual) to the full curvature, so that spacetime is flat away from sources, and there is no interaction among localized masses [5]. Similar considerations hold in presence of a Λ term, except that the exterior now has constant curvature [6]. The same holds for the field strength in (4.2), and non-interaction among charges. Note the counterintuitive property of (4.2) that charges create magnetic, while currents create electric, fields: F^0 is the magnetic field $\epsilon^{ij}F_{ij}$, while F^i is the electric field $\epsilon^{ij}E_j$. Point charges are represented by a current

$$j^\mu = \Sigma e_A u_A^\mu(t)\delta^2(\mathbf{r}_\mu - \mathbf{r}_A(t))\,. \tag{4.3}$$

Note that j^μ is actually a metric-independent contravariant vector density just like F^μ, so the tensor and vector equations are totally independent. Current conservation alone requires the particle worldlines to be continuous (albeit not necessarily future timelike), while covariant conservation constrains any point-like stress tensor to be that of (a sum of) particles [7]. As in gravity, while there is no interaction, the large-scale "geometry" is affected by the configurations: in gravity these are the well-known metrics with conical singularities at the sources, together with their boosted generalizations, as discussed in [5], and similarly, as we now see, for the vector potentials †

The simplest case is a single static charge at the origin,

$$j^0 = e\delta^2(\mathbf{r}) \tag{4.4}$$

which generates a pure magnetic field, $\epsilon^{ij}F_{ij} \sim e\delta^2(\mathbf{r})$ whose vector potential is

$$A_i = \frac{-e}{2\pi}\epsilon^{ij}\partial_j \ln r + \partial_i\alpha\,. \tag{4.5}$$

Clearly, the potential is a superposition of such contributions if there are more static particles. Note that there is no self-force here, since the $\int A_\mu j^\mu$ term vanishes identically. The configuration can be sampled through its Aharonov-Bohm phase, proportional to the sum of the charges. A moving particle will generate an electric field as well; for a single source,

$$F^i = \epsilon^{ij}E_j = \epsilon^{ij}\dot{A}_j = eu^i\delta^2(\mathbf{r} - \mathbf{r}(t)). \tag{4.6}$$

This corresponds to a time-dependent vector potential $A_i(t)$ in $A_0 = 0$ gauge, with step-function behavior.

The Einstein + CS + particle system is now easy to solve; as noted, the vector CS term, being topological, is metric-independent, as is the particle's j^μ, so the combined field equations decouple,

$$G^{\mu\nu} = mu^\mu u^\nu\delta^2(\mathbf{r}), \qquad F^\mu = eu^\mu\delta^2(\mathbf{r}) \tag{4.7}$$

and reduce, for the single static charge, with $u^\mu = \delta^{0\mu}$, to the usual conical space with deficit angle proportional to the source's mass, but independent of any charge properties, and a "conical" vector potential proportional to the total charge but independent of mass, as described by (4.5). The extension to superposition of several static particles is immediate, though there are interesting global geometric complications and limitations on the mass - and perhaps also (color) charge - parameters, and even more for moving particles, despite the absence of true dynamics. Irrespective of the details of generic, distributed, interior sources, the exterior fields are those of a single particle with total mass and charge.

†A more detailed perspective on CS electrodynamics with point charges may be found in [8].

5 Summary

We have discussed two separate problems: the primary one was to obtain a "pure CS" formulation of vector models in $D = 3$ to include the Maxwell action. This required addition of a third-derivative CS-like term. In an AdS background, the same procedure further allowed for a two-CS formulation using the freedom afforded by presence of two mass parameters (m, ℓ^{-1}). Here as in gravity, at either special "chiral" point, one of the two CS terms vanishes. This is also the common point for which TMG and TME equations can be put into one-one correspondence.

Our second topic was that of "pure-CS" in the literal sense of keeping only the CS vector term along with its corresponding gravitational term, the Einstein action (with or without Λ). This two-field system was coupled to charged point masses. Because the two fields are entirely decoupled (CS being topological), the resulting configurations are separate conical metric and vector potential "spaces", with (known) interesting geometric complications in the gravitational sector. The nonabelian vector side should also prove of interest.

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